Episode 4

Newton's Laws: Part 1
Finding forces on particles with known motion

ENGN0040: Dynamics and Vibrations
Allan Bower, Yue Qi

School of Engineering
Brown University

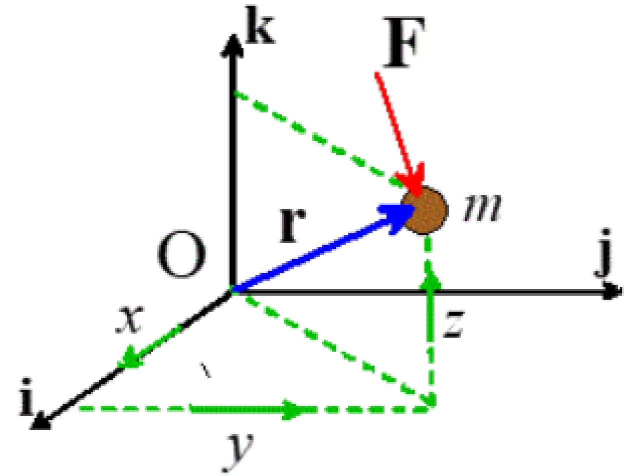
Topics for todays class

Newton's Laws

Calculating forces on particles with known motion

3 Analyzing particle motion using Newtons laws

3.1 Newtons laws



Newton I: $\underline{F} = \underline{0} \Leftrightarrow \underline{v} = \text{const}$

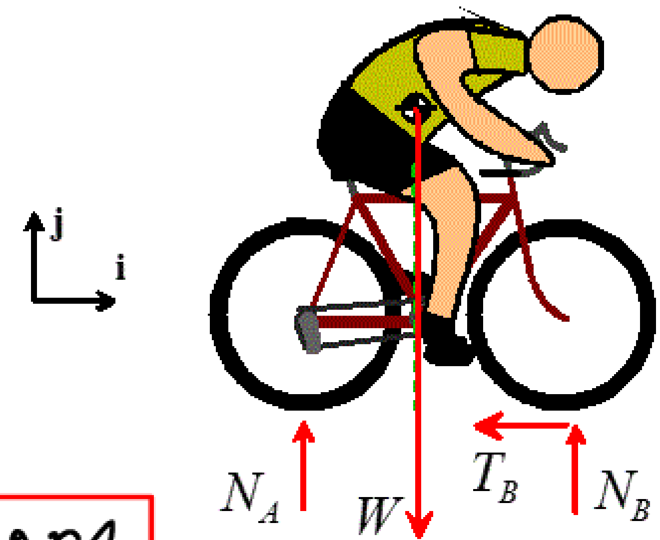
Newton II $\underline{F} = m\underline{a}$

Newton III "Forces come in pairs"
- every action has equal & opposite reaction

A preliminary result from rigid body dynamics

For an object that translates without rotation

$$\sum \underline{r} \times \underline{F} = \underline{0} \text{ about COM}$$



$\boxed{\text{Must take moments about COM in dynamics}}$

General problems

(1) Given $\underline{a}$, find $\underline{F}$

(2) Given $\underline{F}$, find $\underline{a}$, then calculate $\underline{v}$ and $\underline{r}$

3.2 Calculating forces on particles with known motion

Procedure: just like statics

(1) Draw FBD

(2) Find $\underline{a}$ (particle motion)

(3) Solve $\underline{F} = m\underline{a}$ for unknown forces

Illustrate with examples

Review of Friction Laws

(You can skip this part if you already know it!)

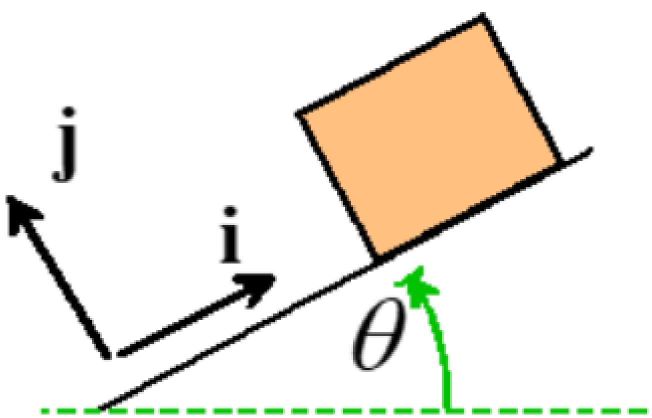
Contact and Friction Forces

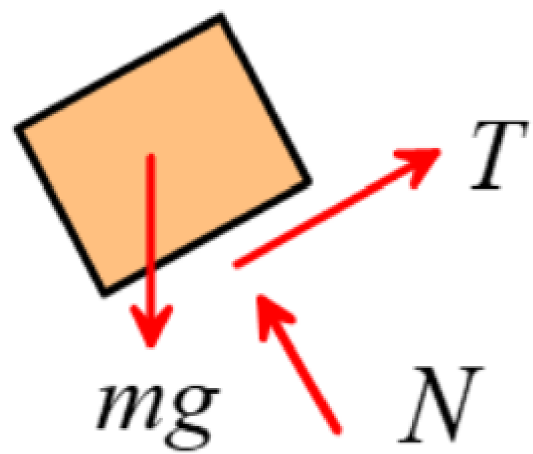
General observations:

1. A normal force always acts at the contact between surfaces
2. A tangential force acts if friction coefficient $\mu > 0$
3. Depending on the loading applied to the bodies in contact:

   The surfaces may remain in contact or separate

   The surfaces may slip or stick



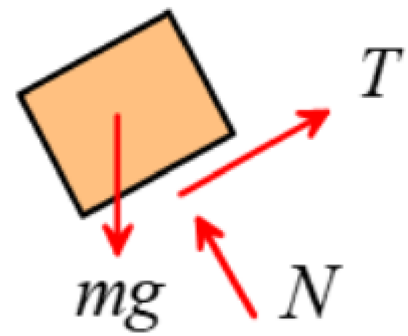


Contact and Friction Forces

The normal force:

Must be repulsive $N \geq 0$.

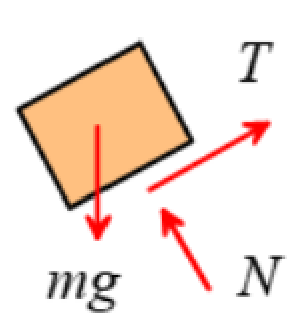
If you find $N < 0$ the surfaces separate

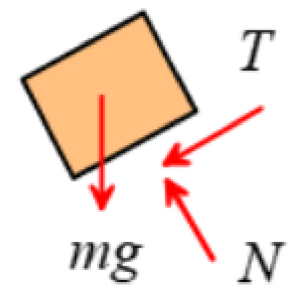


The tangential force (FBD and friction laws):

No Slip: Draw T in either direction.

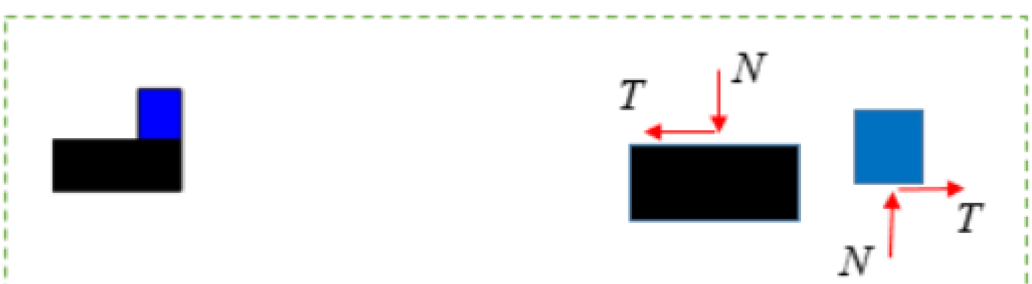
$|T| < \mu N$ (or $|T| < \mu_s N$)

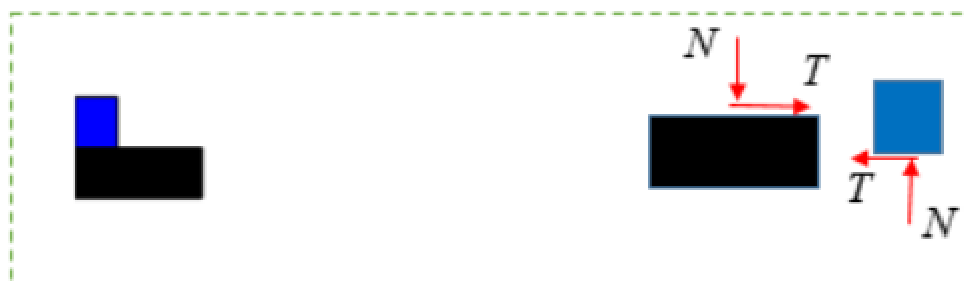




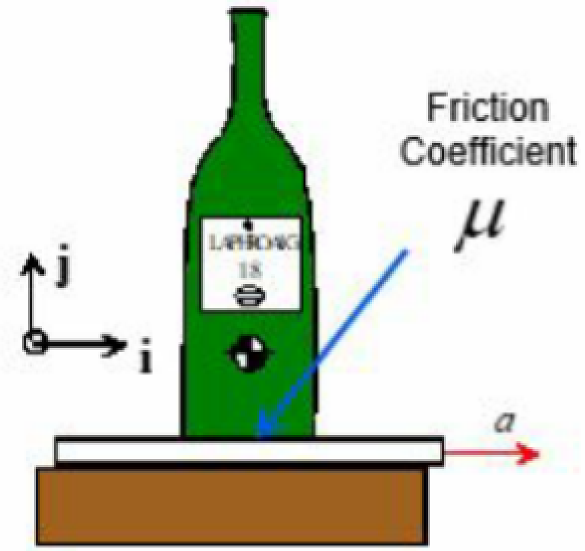
Slip: T must act to resist sliding

$T = \mu N$ (or $T = \mu_k N$)





3.2.1: Example: Tablecloth trick: Find a formula for the critical acceleration that will pull the cloth from under the bottle



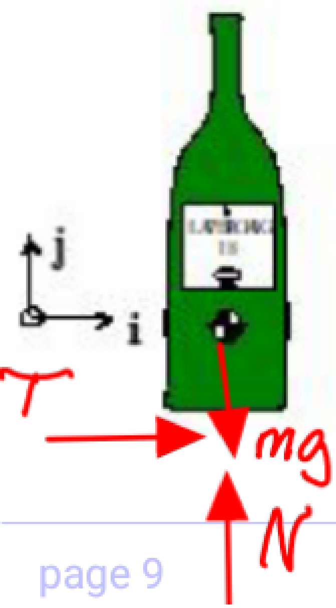
Approach:

(1) We need cloth to slip under bottle

(2) Small $a \Rightarrow$ No slip
Large $a \Rightarrow$ Slip

(3) Assume no slip, find reactions with $F=ma$, then check for slip

FBD



No slip $\Rightarrow$ bottle & cloth have same $\underline{a}$

$$\Rightarrow \underline{a} = a\underline{i}$$

$$\underline{F} = m\underline{a}.$$

$$T\underline{i} + (N-mg)\underline{j} = ma\underline{i}$$

i, j terms on LHS & RHS are equal

$\Rightarrow \quad T = ma$

$\quad\quad N = mg$

Friction law $\quad |T| < \mu N \Rightarrow$ no slip

$\Rightarrow$ For slip $\quad |T| \geq \mu N$

$\Rightarrow \quad |ma| \geq \mu mg$

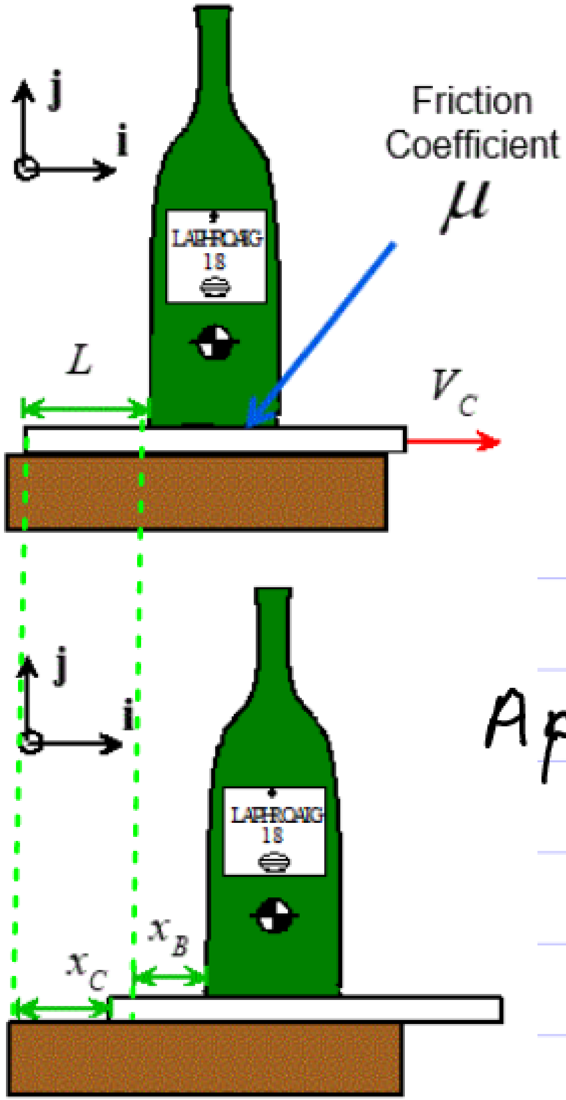
$\boxed{\text{Hence } a \geq \mu g \text{ for trick to work}}$

Typically $\mu \sim 0.1 \Rightarrow \boxed{a > 1 \ m/s^2}$

Tablecloth trick (more realistic): Assume the cloth is pulled with constant speed V_C for t>0

(a) Find a formula for the minimum cloth speed that will pull the cloth from under the bottle

(b) If the cloth is pulled with the minimum speed, find a formula for the distance the bottle has moved when it reaches the edge of the cloth



Note: For small V_0 slip stops before cloth edge reaches bottle => trick fails.

We need to predict motion of cloth & bottle

Approach: (1) Find a_B ($\underset{\sim}{F} = m\underset{\sim}{a}$)

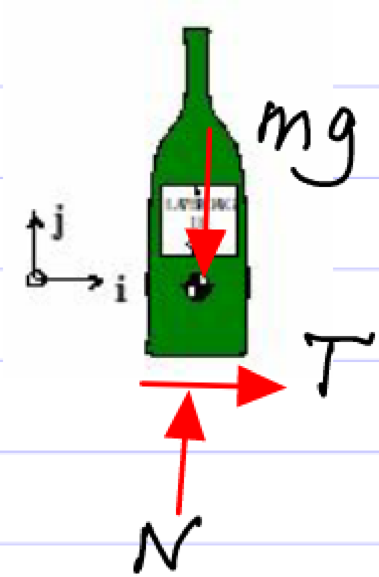
(2) Find V_B, X_B & x_C

(3) Find time t_* when cloth edge reaches bottle

(4) For slip $V_C > V_B$ at time $t = t^*$

(1) Find a_B

FBD



Slip $\Rightarrow$ $T = \mu N$

$\underline{F} = m\underline{a}$ $\Rightarrow$

$$\mu N \underline{i} + (N - mg)\underline{j} = m a_B \underline{i}$$

$\Rightarrow$ $N = mg$

$$a_B = \mu N / m = \mu g$$

(2) Find v_B, x_B, x_c in terms of t

Const accel $\Rightarrow$ $v_B = \mu g t$ $\quad x_B = \frac{1}{2}\mu g t^2$

$$x_c = V_0 t$$

(3) Find time t_* when cloth edge reaches bottle

At $t = t_*$ cloth has travelled dist L further than bottle

$$\Rightarrow \quad x_c = x_B + L$$

$$\Rightarrow \quad V_c t_* = \frac{1}{2}\mu g t_*^2 + L$$

Quadratic eq for t_*, solution

$$t_* = \frac{V_c - \sqrt{V_c^2 - 2\mu g L}}{\mu g}$$

(4) For success $V_c > V_B$ @ $t = t^*$

$\Rightarrow \quad V_c > \mu g t_*$

$\Rightarrow \quad V_c > V_c - \sqrt{V_c^2 - 2\mu g L}$

$\Rightarrow \quad V_c^2 - 2\mu g L > 0$

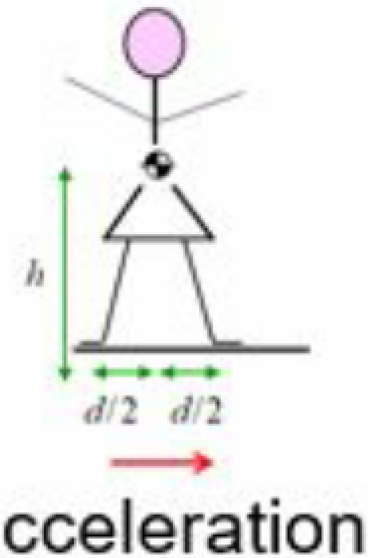
$\Rightarrow \quad \boxed{V_c > \sqrt{2\mu g L}}$

(5) Find x_B @ $t = t_*$ with $V_c = \sqrt{2\mu g L}$

Recall $x_B = \frac{1}{2}\mu g t_*^2$ $\quad t_* = V_c / \mu g$

Hence $x_B = \frac{1}{2}\mu g \left(\frac{\sqrt{2\mu g L}}{\mu g}\right)^2 \Rightarrow \boxed{x_B = L}$!

3.2.2: Example: People mover safety. Recommend a maximum acceleration that will ensure passengers inside a 'people mover' will not tip over



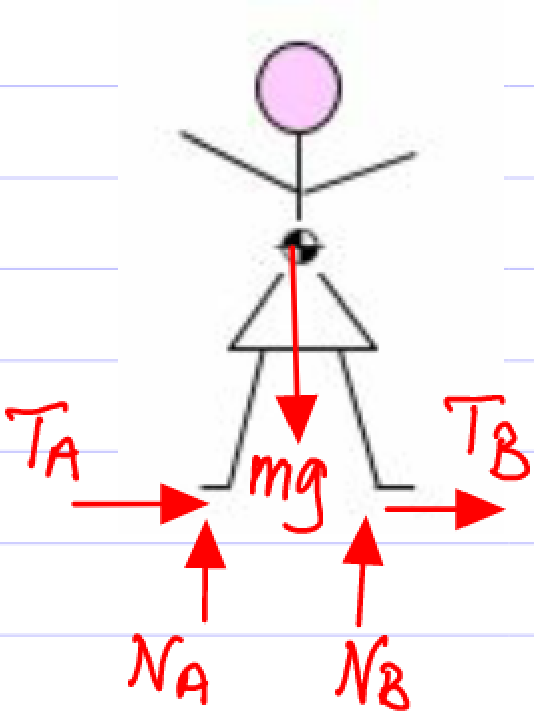
Approach:

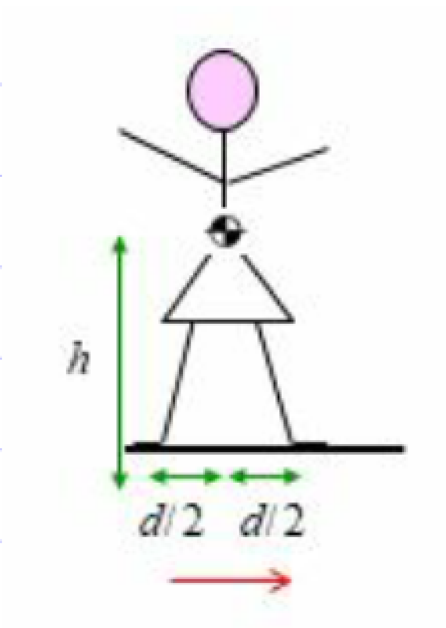
(1) For safety both feet must stay on ground

(2) Small $a \Rightarrow$ OK
Large $a \Rightarrow$ tipping

(3) Use $\mathbf{F} = m\mathbf{a}$ & $\sum \mathbf{r} \times \mathbf{F} = \mathbf{0}$ to find reactions on feet. At critical a for tipping one reaction must be zero. Hence solve for a.

FBD





No slip $\Rightarrow$ $\underline{a} = a\,\underline{i}$

$\underline{F} = m\underline{a}$

$(T_A + T_B)\,\underline{i} + (N_A + N_B - mg)\,\underline{j} = ma\,\underline{i}$

$\sum \underline{r} \times \underline{F} = \underline{0}$ about COM

$\left((T_A + T_B)\,h + N_B \frac{d}{2} - N_A \frac{d}{2}\right)\underline{k} = \underline{0}$

Equations:

$$T_A + T_B = ma \qquad (1)$$

$$N_A + N_B = mg \qquad (2)$$

$$(N_B - N_A)\frac{d}{2} + (T_A + T_B)\,h = 0 \qquad (3)$$

To solve:

Subst (1) in (3) & rearrange

$$N_A - N_B = ma \cdot 2h/d \quad (4)$$
$$N_A + N_B = mg \quad (2)$$

$(4)+(2) \Rightarrow \quad 2N_A = mg + ma \cdot 2h/d$

$(2)-(4) \Rightarrow \quad 2N_B = mg - ma \cdot 2h/d$

$$\Rightarrow \left.\begin{aligned} N_A &= mg/2 + mah/d \\ N_B &= mg/2 - mah/d \end{aligned}\right\} \begin{aligned} &N_A > 0 \quad N_B > 0 \\ &\text{for safety} \end{aligned}$$

$N_B > 0 \Rightarrow \boxed{a < gd/(2h)} \quad d/2h \sim 0.2$

Hence $a < 0.2g$ for safety.

3.2.3: Example: Aircraft in a standard rate turn. At 'standard rate' a 360 degree turn takes 2 mins. Calculate:

The radius of the circular path
The load factor (ratio of lift force to weight)
The angle of bank

Given:

- Aircraft flies at known speed V
- Lift force always acts perpendicular to wings

Approach: Circular motion & $\underline{F} = m\underline{a}$

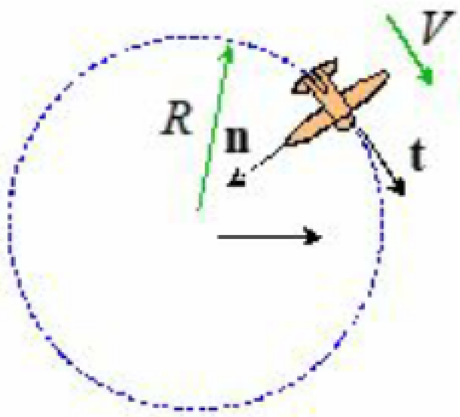
Circular motion $\quad \omega = d\theta/dt \quad V = R\omega$

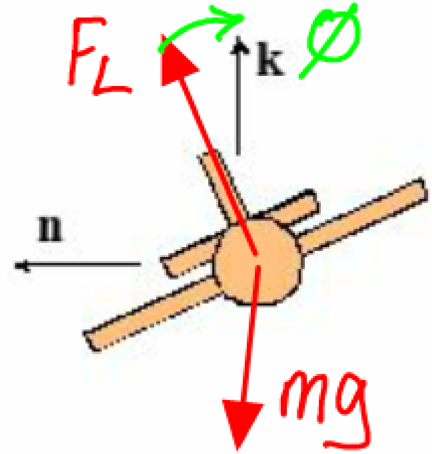
$360^0 = 2\pi$ takes 2 min $\Rightarrow \omega = 2\pi/120$ rad/s

$V = R\omega \Rightarrow \boxed{R = V/\omega = 60V/\pi}$

For small aircraft $V \sim 90$ knots

$\Rightarrow R = 880$ m





Circular motion

$$\underline{a} = \frac{V^2}{R}\underline{n} = V\omega\,\underline{n} \simeq \frac{V\pi}{60}\underline{n}$$

$$F = ma$$

$$F_L \sin\phi\,\underline{n} + (F_L\cos\phi - mg)\,\underline{k} = m\frac{V\pi}{60}\underline{n}$$

$$F_L \sin\phi = mV\pi/60 \qquad (1)$$

$$F_L \cos\phi = mg \qquad (2)$$

$$(1)/(2) \Rightarrow \boxed{\tan\phi \simeq V\pi/60g}$$

$\sim 18^\circ$ @ 90 knots

$$(1)^2 + (2)^2 \Rightarrow F_L^2\underbrace{(\sin^2\phi + \cos^2\phi)}_{=1} = (mg)^2\left(1 + (V\pi/60g)^2\right)$$

$$\Rightarrow \boxed{\frac{F_L}{mg} = \sqrt{1 + \left(\frac{V\pi}{60g}\right)^2}}$$

LOAD FACTOR

~ 1.1 @ 90 knots